

3. 略

$$4. \sum_i \int_{\partial M} p_i dx^i = \sum_{i < j} \int_M \left(\frac{\partial p_j}{\partial x^i} - \frac{\partial p_i}{\partial x^j} \right) dx^i \wedge dx^j$$

$$5. \int_{\partial M} \omega = \int_M \left(\sum_{i=1}^n \frac{\partial p_i}{\partial x^i} \right) dx^1 \wedge \dots \wedge dx^n.$$

6.

$$\text{R.H.S.} = \int_M \left(\sum_{i=1}^n \frac{\partial v}{\partial x^i} dx^i \right) \wedge \left(\sum_{j=1}^n (-1)^{j-1} \frac{\partial u}{\partial x^j} dx^1 \wedge \dots \wedge \widehat{dx^j} \wedge \dots \wedge dx^n \right)$$

$$+ \int_M \left[\frac{\partial}{\partial x^i} \right] \sum_{j=1}^n v (-1)^{j-1} \frac{\partial^2 u}{\partial x^i \partial x^j} dx^1 \wedge \dots \wedge \widehat{dx^j} \wedge \dots \wedge dx^n$$

$$- \int_M \sum_{j=1}^n \frac{\partial u}{\partial x^j} \cdot \frac{\partial v}{\partial x^j} dx^1 \wedge \dots \wedge dx^n$$

$$= \int_M v \cdot \sum_{j=1}^n \frac{\partial^2 u}{\partial x^j \partial x^j} dx^1 \wedge \dots \wedge dx^n = \int_M v \cdot \Delta u dx^1 \wedge \dots \wedge dx^n.$$

7. 只用记: $u|_{\partial M} = 0$, $\Delta u = 0 \Rightarrow u \equiv 0$.

$$\text{由 6, 令 } u = v, \int_M |\nabla u|^2 dx^1 \wedge \dots \wedge dx^n = 0 \Rightarrow \nabla u = 0 \Rightarrow u \equiv 0.$$

8. 6 直接推论

$$9. \int_{\partial S^n} \omega = \int_{\partial S^n} \eta, \int_{S^n} \omega = \int_{\partial S^n} \eta = 0, \text{ since } \partial S^n = \emptyset$$

$$10. d\omega = 0, 0 = \int_M d\omega = \int_{\partial M} \omega = \int_{C_1} \omega + \int_{-C_2} \omega = \int_{C_1} \omega - \int_{C_2} \omega.$$